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An Analytic Model of Cross-correlation In a Bottom Bounce Environment

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PREFACE

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ABSTRACT

Intersensor cross-correlation can be significantly affected by acoustic multipath. An analytic model has been developed to simulate band-limited cross-correlation for an environment in which the propagation is represented as four first-order bottom bounce rays. The "four path" model is intended to be an aid in understanding how cross-correlation is affected by bottom bounce conditions. This memorandum describes the geometry and assumptions of the analytic four path model, presents the supporting mathematics, computes normalized cross-correlations and compares computed normalized cross-correlations with correlations generated by the Generic Sonar Model. The rms error between the two models was found to be less than 0.1 percent for the three test cases examined.

INTRODUCTION

An analytic model has been developed to simulate cross-correlation performance in a bottom bounce environment. This model is based on the following assumptions: 1) band-limited received signals, 2) an isovelocity, lossless ocean, 3) the arrival at each receiver is the sum of four separate bounce paths; bottom, bottom-surface, surface-bottom and surface-bottom-surface, 4) a perfectly reflecting bottom, and 5) a perfectly reflecting surface with 180 degree phase shift. These assumptions, while simplistic, allow the correlation function to be expressed in closed form.

This memorandum describes the model assumptions, geometry, and presents equations for cross-correlation and ray path travel time. Results of a computer implementation are compared with cross-correlation functions generated by the Generic Sonar Model (ref. 1).

THE FOUR PATH MODEL

Figure 1 is a horizontal projection of the model geometry, showing the relative position of source and receivers. Note the definition of the position of receiver 1, and of the bearing angle, θ. L_H is the horizontal projection of the receiver separation, and is defined by

$$L_H = [L^2 - \Delta z^2]^{1/2} \tag{1}$$

where L is the actual receiver separation, and Δz is the depth difference between receivers.

With equation 1, the true range to each receiver may be derived as a function of bearing angle. It can be shown that

$$r_1 = [r^2 + (L_H/2)^2 - L_H \cdot r\cos\theta]^{1/2} \tag{2}$$

$$r_2 = [r^2 + (L_H/2)^2 + L_H \cdot r\cos\theta]^{1/2} \tag{3}$$

where r is the horizontal range to the midpoint of the receiver baseline.

Figure 2 is a vertical projection of the model geometry, showing the bottom bounce rays to each receiver. The source is placed at depth z_0, with the receivers at depths z_1 and z_2, respectively. The time delay between the arrival of the signal at each receiver is defined by

$$\tau_{ij} = \frac{\ell_{1i} - \ell_{2j}}{c} \tag{4}$$

where ℓ_{1i} is the path length from the source to receiver 1, for bounce path i, and ℓ_{2j} is the path length from the source to receiver 2, for

bounce path j. When two subscripts are used it is assumed that the subscript i is associated with receiver 1, and subscript j with receiver 2. With this notation the intra-path delay shown in Figure 2 is denoted τ_{ij}, being the delay between the bottom bounce path at receiver 1 and the bottom bounce path at receiver 2.

The total cross-correlation may now be written as the arithmetic sum of the individual path correlations. For band-limited signals, the signal bandwidth is defined as

$$B = f_H - f_L \tag{5}$$

where f_H and f_L are the upper and lower cutoff frequencies. Similarly, the center frequency is defined as

$$\omega_o = 2\pi \frac{f_H + f_L}{2} \tag{6}$$

The correlation of paths i and j is then written as

$$\chi_{ij}(\tau) = A_{ij}\cos[\omega_o(\tau-\tau_{ij})]\frac{\sin[\pi B(\tau-\tau_{ij})]}{\pi B(\tau-\tau_{ij})} \tag{7}$$

The amplitude of each correlation is the product of the individual path amplitudes, or

$$A_{ij} = a_{1i}a_{2j} \tag{8}$$

Each of the individual path amplitudes is a function of the initial signal amplitude, Ao, and spherical spreading loss only. The amplitude, in decibels, of the signal at receiver k from bounce path i, is:

$$a_{ki}(dB) = 20\mathrm{Log}_{10}A_o - 20\mathrm{Log}_{10}\ell_i \tag{9}$$

where ℓ_i is the length of the ith bounce path.

The path lengths are easily derived for an isovelocity ocean and are functions of depth and the tangent of the arrival angles. A

vertical projection of the bottom bounce path is shown in Figure 1. It can be shown that the tangent of the arrival angle at receiver i, is

$$\tan\phi_{Bi} = \frac{2d - (z_o + z_i)}{r_i} \qquad (10)$$

and the total bottom bounce path length is

$$\ell_{Bi} = [(d-z_o)^2 + (\frac{d-z_o}{\tan\phi_{Bi}})^2]^{1/2} + [(d-z_i)^2 + (\frac{d-z_i}{\tan\phi_{Bi}})^2]^{1/2} \qquad (11)$$

where

- d = water column depth in meters
- z_o = source depth
- z_i = depth of receiver i
- r_i = horizontal range to receiver i

Similarly, for the bottom-surface bounce path,

$$\tan\phi_{BSi} = \frac{2d - (z_o - z_i)}{r_i} \qquad (12)$$

and

$$\ell_{BSi} = [(d-z_o)^2 + (\frac{d-z_o}{\tan\phi_{BSi}})^2]^{1/2} + [d^2 + (\frac{d}{\tan\phi_{BSi}})^2]^{1/2}$$

$$+ [z_i^2 + (\frac{z_i}{\tan\phi_{BSi}})^2]^{1/2} \qquad (13)$$

The surface-bottom bounce path,

$$\tan\phi_{SBi} = \frac{2d + (z_o - z_i)}{r_i} \tag{14}$$

and

$$\ell_{SBi} = [(z_o{}^2 + (\frac{z_o}{\tan\phi_{SBi}})^2]^{1/2} + [d^2 + (\frac{d}{\tan\phi_{SBi}})^2]^{1/2}$$

$$+ [(d-z_i)^2 + (\frac{d-z_i}{\tan\phi_{SBi}})^2]^{1/2} \tag{15}$$

The surface-bottom-surface path,

$$\tan\phi_{SBSi} = \frac{2d + (z_o + z_i)}{r_i} \tag{16}$$

and

$$\ell_{SBSi} = [z_o{}^2 + (\frac{z_o}{\tan\phi_{SBSi}})^2]^{1/2} + 2\cdot[d^2 + (\frac{d}{\tan\phi_{SBSi}})^2]^{1/2}$$

$$+ [z_i{}^2 + (\frac{z_i}{\tan\phi_{SBSi}})^2]^{1/2} \tag{17}$$

COMPUTER IMPLEMENTATION

The equations presented in the previous section have been implemented on the Code 33 VAX system under the name DRBO:[HAUCK]4PATH. Each bounce path length, and arrival angle is calculated by a subroutine named DEL**, where ** indicates the type of path. For example, DELB calculates the path length for the bottom bounce path. Attenuation for each of the eight separate travel paths is then calculated by the subroutine ATTN. Sixteen inter-path correlations are calculated and summed, with the resulting time series normalized by the sum of the four intra-path energies. Each intra-path energy, where i=j, is defined by

$$E_{ij} = \sum_{k=1}^{N} \frac{1}{[a_{ik}^2 + a_{jk}^2]^{1/2}} \tag{18}$$

where N is the total number of samples in the observation window. The total energy is then the sum of the four intra-paths, or

$$E_N = \sum_{i=1}^{4} E_{ii} \tag{19}$$

The final result is then plotted using a general-purpose plotting routine, ALLPLOT, and written in a Generic Sonar Model compatible output file. Alternatively, intermediate values of delay time, range and phase may be written to the user's default output device.

Testing of the model was accomplished by comparison with results produced by the GSM. The GSM computes the cross-correlation via an FFT of the synthesized received cross-spectrum (ref. 1). Three test cases were examined. The parameters common to all three cases are defined in Table I.

Table I - Parameters Common to All the Three Test Cases

Sound Speed	= 1500 m/s
Water Depth	= 1000 m
Source Depth	= 25 m
Range (Source to center of receiver baseline)	= 1000 m
f_L	= 200 Hz
f_H	= 700 Hz
Receiver 1 depth	= 50 m

In the first case, the source is broadside (0 deg) to the receiver baseline, and receiver 2 is at the same depth as receiver 1. (The sample runstream required to run the GSM for this case is given in Appendix I. The interested reader can observe in the runstream the appropriate statements requried to model a four path isovelocity, and

zero attenuation medium). Figure 3 presents for this case the cross-correlations obtained by both the analytic four path model and by the GSM. The results are indistinguishable by visual comparison. In the second case the source is again at 0° bearing, while receiver 2 is now at a depth of 60 m. Figure 4 shows the comparison of the two models for the second case. In the third case, the source bearing is now 45°. Figure 5 shows the comparison of the two models for the third test case.

The cross-correlation time series produced by each model were compared numerically using the program 4ERR. For a given case, this program computes the rms amplitude error between the analytic and GSM correlations. In each of the cases examined, the rms comparison error was less than 0.001, as might be expected by visual inspection of Figures 3-5. Also note that the placement of the correlation peaks, in time, agrees to within the resolution given by the sampling rates used.

SUMMARY AND CONCLUSION

To summarize, this memorandum describes an analytic model designed to predict the cross-correlation of two receivers in a four path, bottom bounce environment. By comparing the output of the model with the GSM, an FFT-based computation and the analytic solution may be compared. For each of the three test cases examined, no significant differences between the correlations obtained by the two models were observed.

In conclusion, comments addressing the limits of the analytic model are in order. The entire model derivation is based on several assumptions, restricting the type of propagation, sound speed profile, boundary interaction, attenuation and the form of the received signals. The sound speed profile is assumed constant. While this assumption implies a direct path between source and receivers, the model assumes bottom bounce propagation. The only allowed paths are the bottom bounce, bottom-surface, surface-bottom, and the surface-bottom-surface. The ocean bottom is modeled as perfectly rigid, with reflection coefficient of unity. The ocean surface is assumed perfect pressure release, with reflection coefficient of -1. The ocean is modeled with no attenuation, the only losses caused by spherical spreading of the transmitted signal. Lastly, the received signal is assumed band-limited, with constant amplitude across the signal bandwidth.

By employing these limitations, it is possible to derive an exact solution for the cross-correlation function. The intent is not to replace more realistic computer models, such as the GSM, but to provide a model which may quickly predict, from a closed form analytic representation, correlation performance in an idealized, bottom bounce ocean. The model thus can be utilized to provide physical insight into the effects of bottom bounce multipath on the correlation process.

REFERENCES

1. H. Weinberg, Generic Sonar Model, NUSC TD 5971C, 15 Dec 1981 (UNCLASSIFIED).

APPENDIX I

```
COMMENT TABLE
RUNSTREAM FOR 4PATH COMPARISON
LOSSLESS, ISOVELOCITY OCEAN
EQUAL DEPTH RECEIVERS
EOF
RANGE UNITS = M
TIME UNITS = MS
TIME MINIMUM = -150.0 MS
BOTTOM DEPTH = 1000 M
OCEAN SOUND SPEED TABLE
M          M/S
0.         1500.0
250.       1500.0
1000.      1500.0
EOF
RADIUS OF CURVATURE = 999999999 KM
OCEAN SOUND SPEED MODEL = LINEAR
DEPTH UNITS = M
VELOCITY UNITS = M/S
SOURCE DEPTH = 25 M
VOLUME ATTENUATION MODEL = FRQTBL
VOLUME ATTENUATION TABLE = 0 DB/KM
SURFACE REFLECTION COEFFICIENT MODEL = TABLE
SURFACE REFLECTION COEFFICIENT TABLE = 0 DB
BOTTOM REFLECTION COEFFICIENT MODEL = TABLE
BOTTOM REFLECTION COEFFICIENT TABLE = 0 DB
FREQUENCY MINIMUM = 200 HZ
FREQUENCY MAXIMUM = 700 HZ
FREQUENCY INCREMENT = 250 HZ
RANGE MINIMUM = 1000. M
RANGE MAXIMUM = 1000. M
RANGE INCREMENT = 1000. M
SOURCE LEVEL TABLE
HZ         DB
1.         -300.
199.       -300.
200.        100.
700.        100.
701.       -300.
3100.      -300.
EOF
FILTER EQUALIZER MODEL = TABLE
FILTER EQUALIZER TABLE
HZ         DB
100.       -300.
199.       -300.
200.        0.
450.        0.
700.        0.
701.       -300.
3100.      -300.
EOF
```

```
TRANSMITTER BEAM PATTERN MODEL = TABLE
TRANSMITTER BEAM PATTERN TABLE
DEG        DB
-90.        0.
-11.        0.
-5.        -300.
 5.        -300.
 11.        0.
 90.        0.
EOF
TARGET DEPTH = 50 M
EIGENRAY FILE = TE1
COMPUTE EIGENRAYS
SORT EIGENRAYS
PRINT EIGENRAYS
EIGENRAY FILE = TE2
TARGET DEPTH = 50 M
COMPUTE EIGENRAYS
SORT EIGENRAYS
PRINT EIGENRAYS
RECEIVER BEAM PATTERN MODEL = TABLE
RECEIVER BEAM PATTERN TABLE
DEG        DB
-90.       -300.
-65.       -300.
-64.5         0.
-63.5         0.
-63.       -300.
  0.       -300.
 61.5      -300.
 62.2         0.
 63.5         0.
 64.       -300.
 90.       -300.
EOF
BEARING ANGLE = 0 DEG
ARRAY SEPARATION TABLE = 100 M
NOISE CORRELATION DISTANCE = 5 M
AMBIENT NOISE SPECTRA MODEL = TABLE
AMBIENT NOISE SPECTRA TABLE = -200 DB
FREQUENCY MAXIMUM = 3100 HZ
CORRELATION FILE = CT1
EIGENRAY FILE = TE1
RESET CROSS-CORRELATION COEFFICIENT
COMPUTE CROSS-CORRELATION COEFFICIENT
RECEIVER BEAM PATTERN TABLE
DEG        DB
-90.       -300.
-65.       -300.
-64.5         0.
-63.5         0.
-63.       -300.
  0.       -300.
 61.5      -300.
```

```
 62.2        0.
 63.5        0.
 64.      -300.
 90.      -300.
EOF
EIGENRAY FILE = TE2
COMPUTE CROSS-CORRELATION COEFFICIENT
END
```

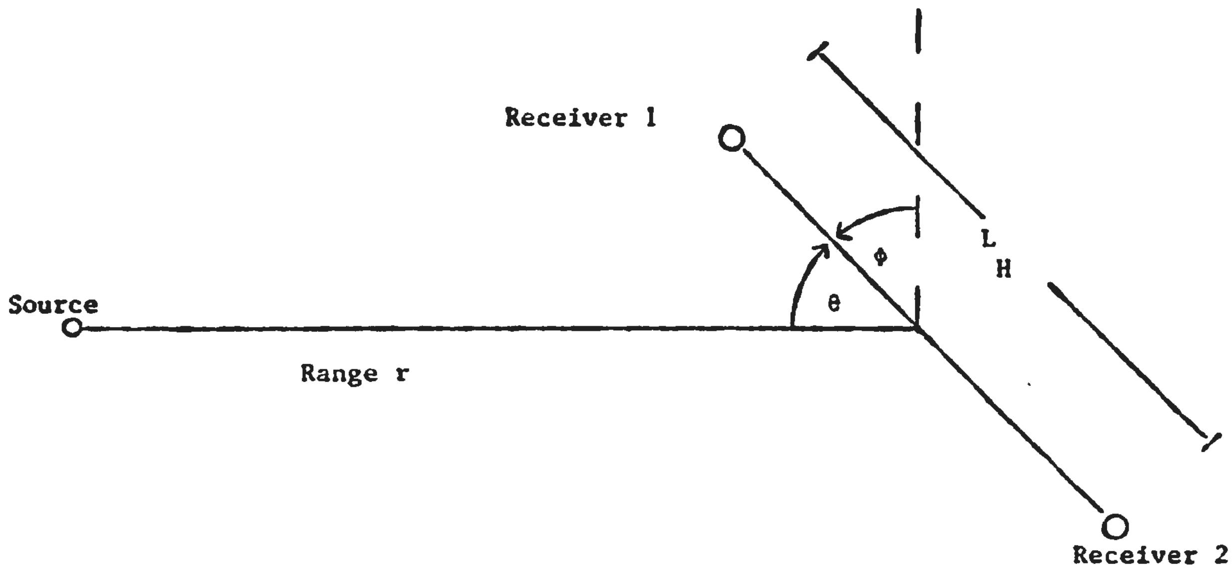

Figure 1 - Horizontal Projection of Model Geometry; bearing angle θ is used by 4 PATH for range calculation, ϕ is the angle input to the program.

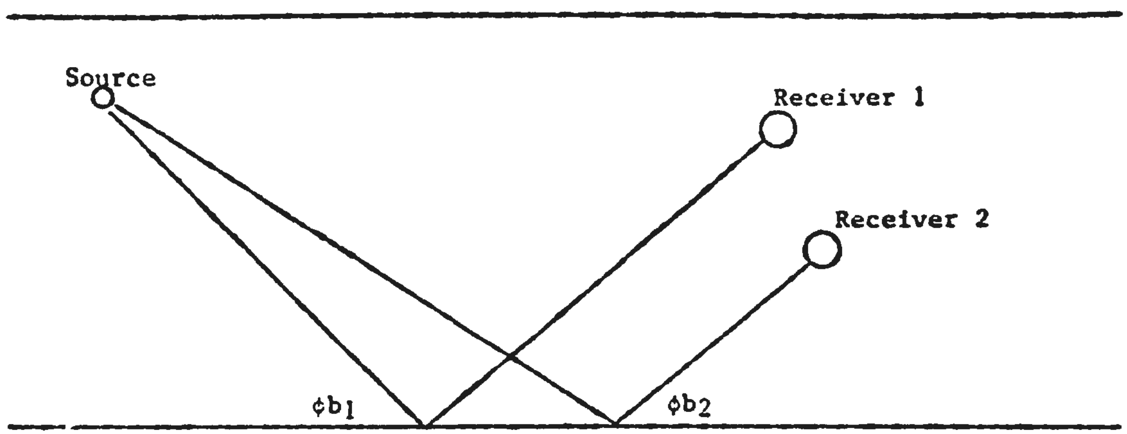

Figure 2 - Vertical Projection of Model Geometry, showing Bottom Bounce Angles ϕb_1 and ϕb_2.

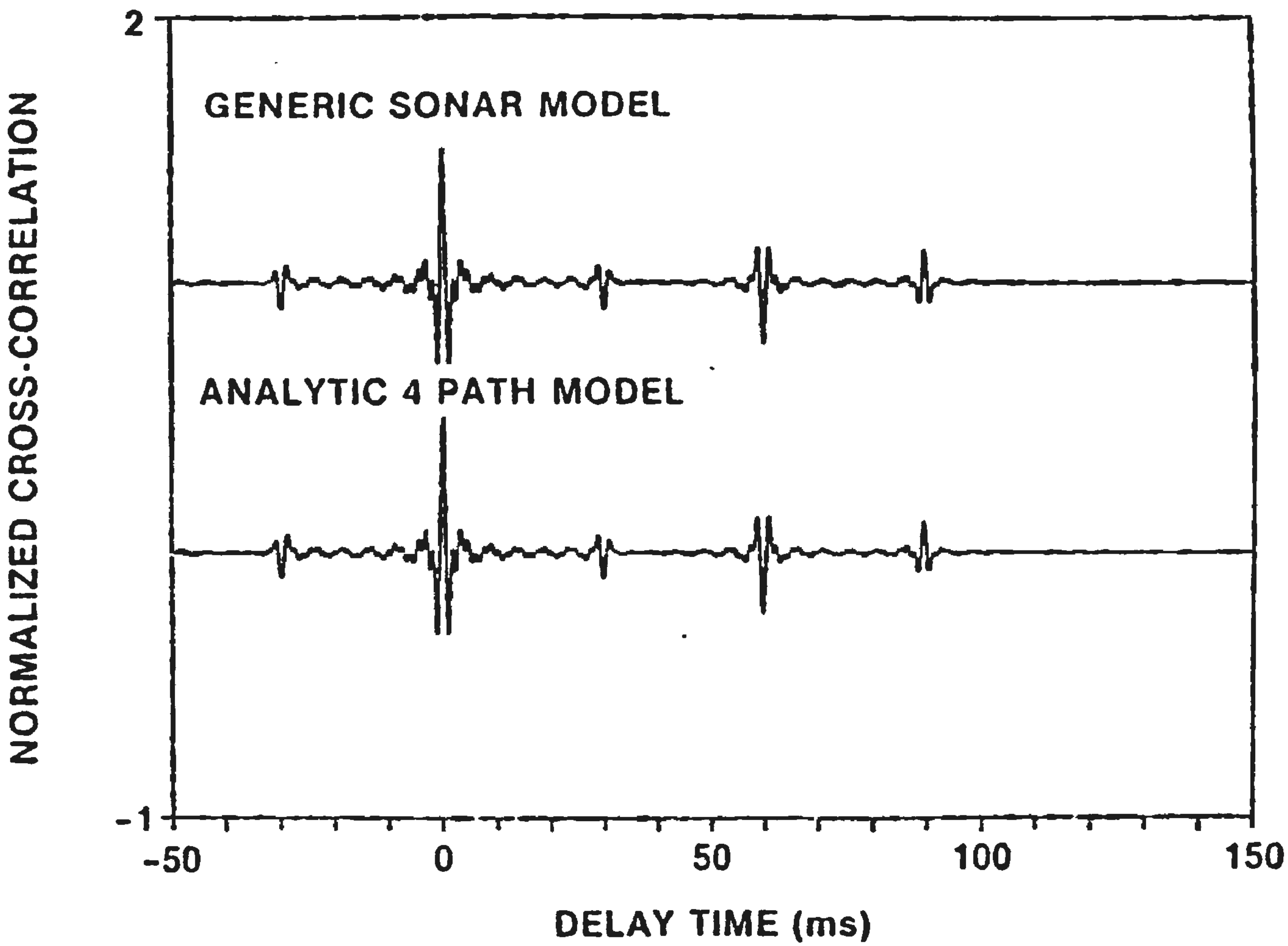

Figure 3 - Generic Sonar Model and Analytic Four Path Model Cross-correlation Comparison where Source Bearing Angle is 0° and Receivers are at Equal Depth.

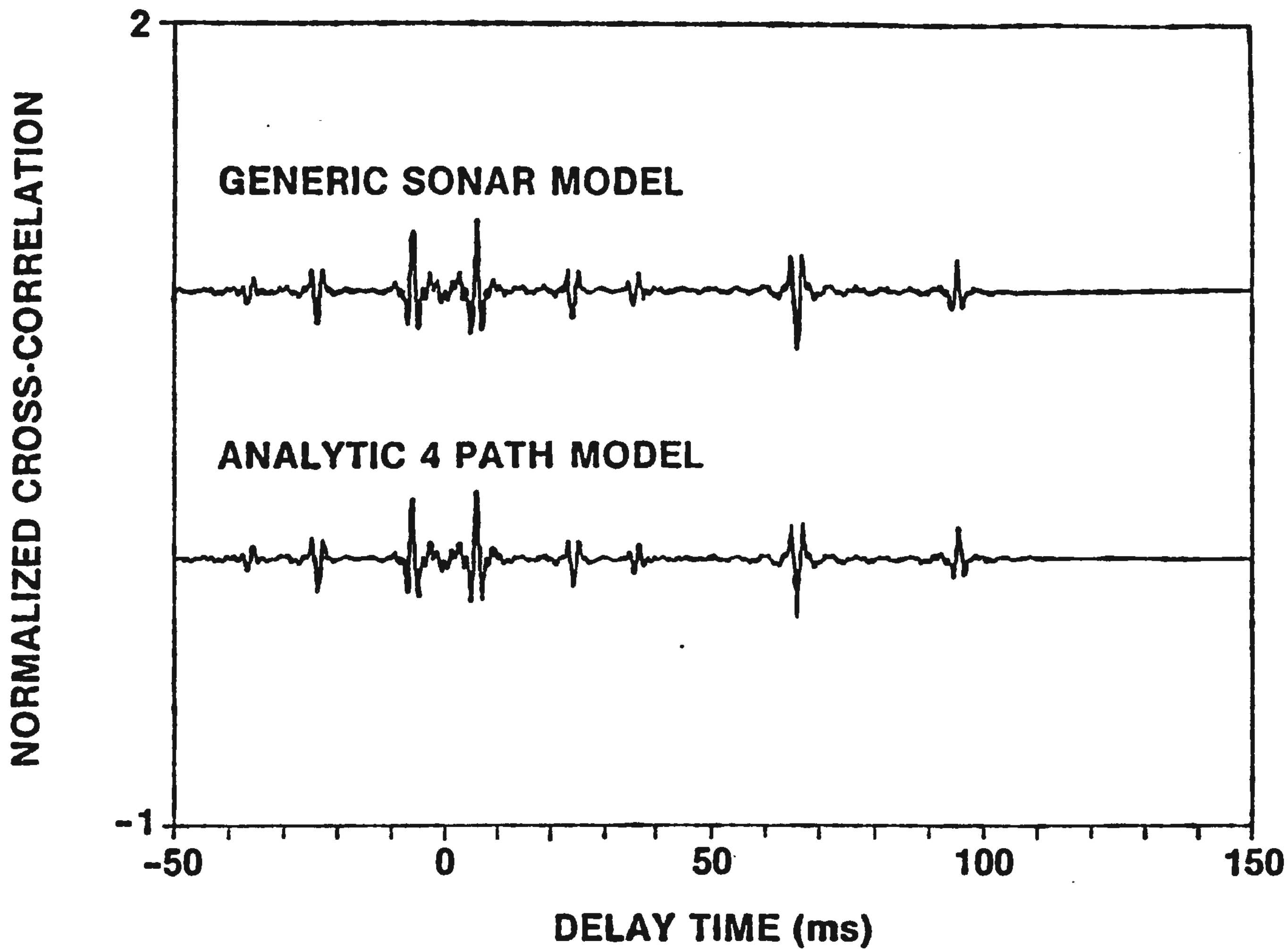

Figure 4 - Generic Sonar Model and Analytic Four Path Model Cross-correlation Comparison where the Source Bearing Angle is 0°, and Receiver 2 is at a Depth of 60 meters.

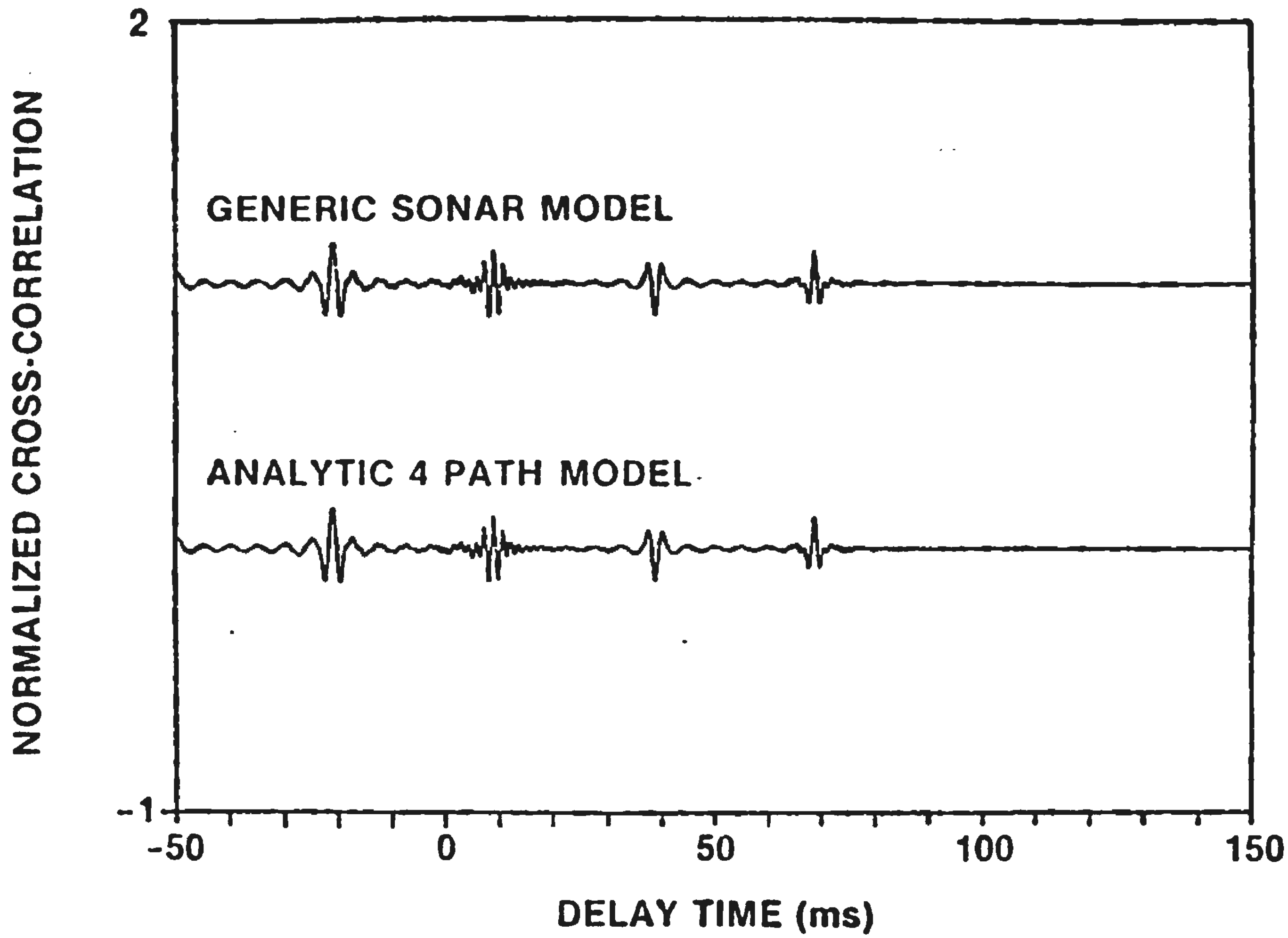

Figure 5 - Generic Sonar Model and Analytic Four Path Model Cross-correlation Comparison where the Source Bearing Angle is 45°, and Receivers are at Equal Depth.

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